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Modeling and simulation of a thermostatic mixer with an anti-scalding or anti-cold system

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Abstract

This paper presents the detailed modeling of a thermostatic mixer for domestic use with an anti-scalding or anti-cold device, which is operated by the temperatures and pressures of the hot and cold streams entering the mixer. The response time of the anti-scalding/cold device is shorter than the response time of the temperature adjustment system, which is based on a bulb that expands or contracts when temperature changes, thus leading to the immediate closing of the mixer inputs if any of the hot or cold streams is absent. This can be critical when the mixer is used by people with long reaction times. The modeling describes the location of all the mixer mobile parts, as well as the temperature evolution along the various identifiable chambers in the mixer. The model is numerically implemented on a well-known modeling tool and is simulated with an implicit Runge–Kutta based method, suitable to the numerical integration of stiff systems. Results for the transient and steady-sate operations are presented, which are relevant both in what concerns the output mixture temperature and the main operating characteristics of the mixer.

Keywords: Thermostatic mixer; Anti-scalding/cold system; Safe operation; Modeling and simulation; Response time analysis

1. Introduction

Thermal equipment is becoming increasingly sophisticated due to consumers' demands when comfort and safety are concerned. Usual thermostatic mixers include a bulb that senses the outlet temperature and then adjusts accordingly to a desired temperature value, specified by the user. This system works well, but it shows up some limitations if sudden changes occur in the hot or cold streams, due to the fact that it has a response time of some seconds. Under these conditions, children, elderly or handicapped people can be subjected, during some seconds, to hot or to cold water streams. The same can be true if accidents occur with people in the bathroom. In order to prevent these possibilities, an anti-scalding/cold system can be incorporated in the mixer, with a very short response time, which closes the water entries to the mixer if any of the hot or cold streams are absent. Regarding the thermostatic part of the mixer, there is a great similarity to that of the usual thermostatic mixers. The model presented can be used for both cases of thermostatic mixers with or without the anti-scalding/cold device, as the action of the anti-scalding/cold device can be easily inhibited in the model.

To the knowledge of the authors, only simple thermostatic mixers have been studied [1], as well as hot water supply systems [2,3], and there are no studies concerned with the modeling and simulation of such devices including the anti-scalding/cold system. Some studies can be found in the literature but they deal with thermostatic valves associated to refrigeration or air conditioning systems, as well as with the use of shape memory materials for temperature control in thermostatic mixers [4,5].

To model a system like a thermostatic mixer it is necessary to take into account the position of all its mobile parts, as well as the temperature evolution of the water along the entire valve, from the inlets to the output. This includes the evaluation of the pressure of the fluid streams along the valve, and how these pressures act on the anti-scalding/cold system, which is linked to the involved flow rates and the particular geometry of the fluid pathways. The proposed anti-scalding/cold system is self-operated by the pressures of the hot and cold streams. The user

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Nomenclature

A	area of fluid passage	m^2
b	friction coefficient	kg/s
c	specific heat	J/(kg K)
C_d	discharge coefficient	
d	diameter	m
e	thickness	m
E	total energy	J
f	fraction of perimeter used for passage	
h	convection heat transfer coefficient	$W/(m^2 K)$
h	specific enthalpy	J/kg
$\dot{H}$	convection heat transfer rate	W
k	thermal conductivity	W/(m K)
L	length	m
m	mass	kg
$\dot{m}$	mass flow rate	kg/s
P	pressure	Pa
Pr	Prandtl number	
$\dot{Q}$	volumetric flow rate	m^3/s
R	dimensionless parameter	
Re	Reynolds number	
t	time	s
T	temperature	°C
v	velocity	m/s
V	volume	m^3
x	space co-ordinate	m

Greek symbols

α	expansion factor for the bulb	m/K
β	compressibility coefficient	Pa
γ	factor, for the desired temperature value	m/K
Δ	difference value	
μ	dynamic viscosity of the fluid	kg/(m s)
ρ	density	kg/m^3

Subscripts

a	anti-scalding/cold device
amb	environment value
atm	atmospheric value
b	bulb
c	cold
cv	control volume
d	desired temperature value
ext	exterior
f	fluid
h	hot
in	inlet
m	averaged value, for material
max	maximum value
min	minimum value
out	outlet
r	rod
ss	steady state
0	reference value

can specify both the output flow rate and the desired mixed temperature. Energy conservation analysis gives the time evolution of the temperature in the different chambers of the valve, and the time evolution of the output mixture temperature can be predicted. Such a model includes also the heat exchanges between the valve and the environment.

The steady-state model infers from the transient model whose analysis leads to important conclusions, both regarding the output mixture temperature and the main operating characteristics of this kind of device. The paper presents results for the steady-state situation, the complete implementation of the model and results of the unsteady mixer simulation. Results provided by the complete unsteady model can be useful in many ways. They can help the designer of a thermostatic mixer, in order to anticipate its behavior and performances. Also very important is the analysis of the output temperature associated with intense transient regimes, due to intense heating, intense cooling, rapid operation by the user of the temperature and flow rate control levers, or sudden absence of the inlet hot or cold streams. Response time of the mixer can be analyzed, as a combination of the mixer (bulb) system, with a response time of some seconds, and of the anti-scalding/cold system with a shorter response time.

In what concerns model implementation and numerical simulation issues, a modeling language called Modelica [6] was used to implement the mixer's model. As an equation based non-causal modeling language, Modelica allows that the equations may be introduced directly. The Matlab [7] and Simulink [8] packages were used to generate the thermostatic mixer inputs, run the simulations and monitoring the results.

2. Modeling

The thermostatic mixer under consideration is schematically represented in Fig. 1. It consists mainly of a mixing chamber, where the streams of hot and cold water are mixed in order to obtain the desired outlet temperature, and a pressure-driven inner rod with three assembled cylinders, which close the mixer inlets if the hot or the cold inlet pressures decrease. The later one is the anti-scalding/cold system, as the outlet mass flow rate is reduced to zero if any of the hot or cold streams is reduced to or closely to zero. Referring to Fig. 1, if, for instance, the inlet pressure of the cold stream decreases relatively to the inlet pressure of the hot stream, then the pressure in chamber 4 becomes lower than the pressure in chamber 1 and the anti-scalding/cold device moves to the right and decreases the hot water inlet into the mixer. The inverse situation occurs if a decrease is observed on the inlet pressure of the hot stream and the anti-scalding/cold device acts to close the inlet cold water into the mixer. This self-actuated anti-scalding/cold device, driven by pressure, has

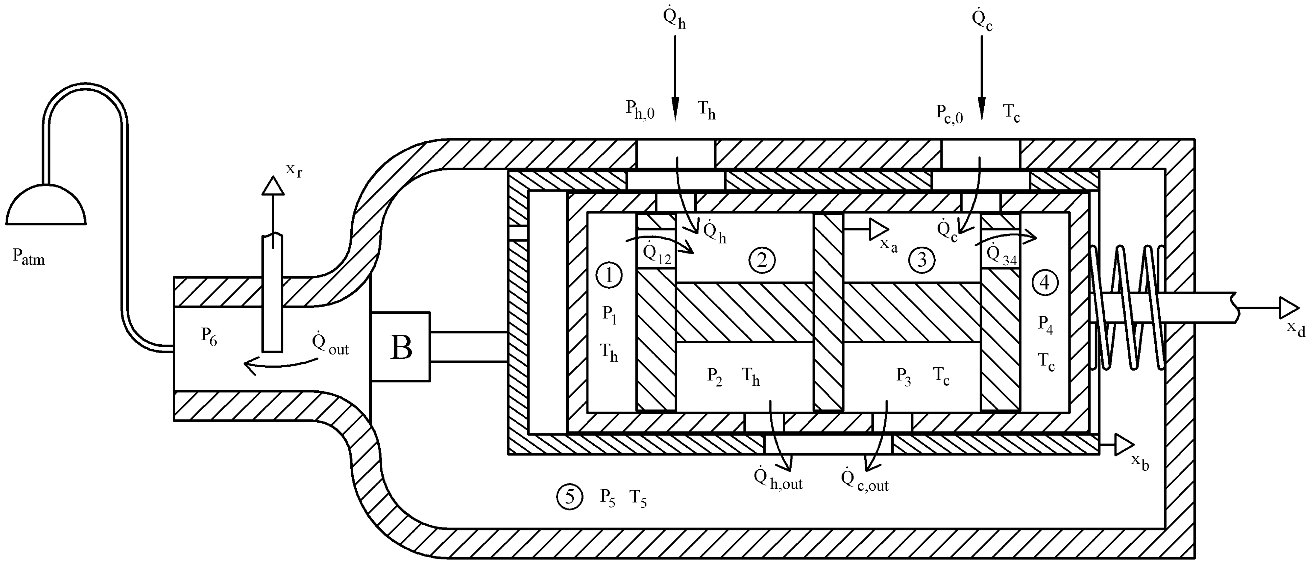

Fig. 1. Schematic representation of the thermostatic mixer.

a response time considerably shorter that the bulb driving the thermostatic action of the mixer. The mixing of the hot and cold streams is made inside the mixer and the relative contribution of each stream is conditioned by the position of the mixing chamber element. This position is controlled both by a bulb, which affects the position of its outlet element proportionally to the temperature of the mixed water, and by a lever used by the operator to settle on the desired outlet water temperature, T_{d}. The outlet flow rate is set by the operator through the position of another lever, x_{r}. The mass conservation principle, the energy conservation principle, and the Newton's Second Law of Motion are used to relate the variables that describe the behavior of the thermostatic mixer. Additional information to derive the full solution is borrowed from the pressure drop relations and from a specific relation describing the bulb behavior.

2.1. Mass conservation equation

For each of the volumetric chambers defined inside the mixer, the mass conservation equation reads [9]

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \dot{m}_{\mathrm{in}} - \dot{m}_{\mathrm{out}} \tag{1}$$

If the fluid is assumed to be slightly compressible, with a high compressibility coefficient evaluated as $\beta = \rho(\mathrm{d}P/\mathrm{d}\rho)_T$, the mass conservation equation for each chamber becomes [10]

$$\frac{\mathrm{d}V}{\mathrm{d}t} + \frac{V}{\beta}\frac{\mathrm{d}P}{\mathrm{d}t} = \dot{Q}_{\mathrm{in}} - \dot{Q}_{\mathrm{out}} \tag{2}$$

where $\dot{Q}$ is the volumetric flow rate and V is the chamber's volume.

Volumes of chambers 2 and 3 are equal and constant, and making use of Fig. 2 they are given by

$$V_2 = V_3 = L_2\pi\left(d_{\mathrm{a}}^2 - d_{\mathrm{r}}^2\right)/4 \tag{3}$$

while volume V_5 is also constant, and given by the particular construction details of the internal body of the mixer.

The mass conservation equations for chambers 2, 3 and 5 give, respectively,

$$\frac{V_2}{\beta}\frac{\mathrm{d}P_2}{\mathrm{d}t} = \dot{Q}_{\mathrm{h}} + \dot{Q}_{12} - \dot{Q}_{\mathrm{h,out}} \tag{4a}$$

$$\frac{V_3}{\beta}\frac{\mathrm{d}P_3}{\mathrm{d}t} = \dot{Q}_{\mathrm{c}} - \dot{Q}_{34} - \dot{Q}_{\mathrm{c,out}} \tag{4b}$$

$$\frac{V_5}{\beta}\frac{\mathrm{d}P_5}{\mathrm{d}t} = \dot{Q}_{\mathrm{h,out}} + \dot{Q}_{\mathrm{c,out}} - \dot{Q}_{\mathrm{out}} \tag{4c}$$

where it is to be noted that $\dot{Q}_{12} > 0$ if $P_1 > P_2$ and that $\dot{Q}_{12} < 0$ otherwise, and that $\dot{Q}_{34} > 0$ if $P_3 > P_4$ ad that $\dot{Q}_{34} < 0$ otherwise. It is always $\dot{Q}_{\mathrm{out}} \geqslant 0$ as it is always $P_5 \geqslant P_6$. The volumetric flow rates $\dot{Q}_{12}$ and $\dot{Q}_{34}$ are related to the change of position of the anti-scalding/cold device, and the volumes of chambers 1 and 4 depend on that position, given by distance x_{a}, and the volume of chambers 1 and 4 is obtained from

$$V_1 = L_1\pi d_{\mathrm{a}}^2/4 \tag{5a}$$

$$V_4 = L_4\pi d_{\mathrm{a}}^2/4 \tag{5b}$$

where lengths L_1 and L_4 are obtained as

$$L_1 = L_{1,0} + x_{\mathrm{a}} - x_{\mathrm{d}} \tag{6a}$$

$$L_4 = L_{4,0} + x_{\mathrm{d}} - x_{\mathrm{a}} \tag{6b}$$

Lengths $L_{1,0}$ and $L_{4,0}$ are the reference values for L_1 and L_4, respectively. From Eqs. (6a) and (6b) it can be obtained that $L_1 + L_4 = L_{1,0} + L_{4,0} = \text{constant}$.

The mass conservation equations for chambers 1 and 4 can be obtained, in the form of Eq. (2), respectively, as

$$\pi\frac{d_{\mathrm{a}}^2}{4}\left[(v_{\mathrm{a}} - v_{\mathrm{d}}) + (L_{1,0} + x_{\mathrm{a}} - x_{\mathrm{d}})\frac{1}{\beta}\frac{\mathrm{d}P_1}{\mathrm{d}t}\right] = -\dot{Q}_{12} \tag{7a}$$

$$\pi\frac{d_{\mathrm{a}}^2}{4}\left[(v_{\mathrm{d}} - v_{\mathrm{a}}) + (L_{4,0} + x_{\mathrm{d}} - x_{\mathrm{a}})\frac{1}{\beta}\frac{\mathrm{d}P_4}{\mathrm{d}t}\right] = \dot{Q}_{34} \tag{7b}$$

where v_{a} and v_{d} are the velocities $\mathrm{d}x_{\mathrm{a}}/\mathrm{d}t$ and $\mathrm{d}x_{\mathrm{d}}/\mathrm{d}t$, respectively. x_{d} is the position of the lever left to the user to set the

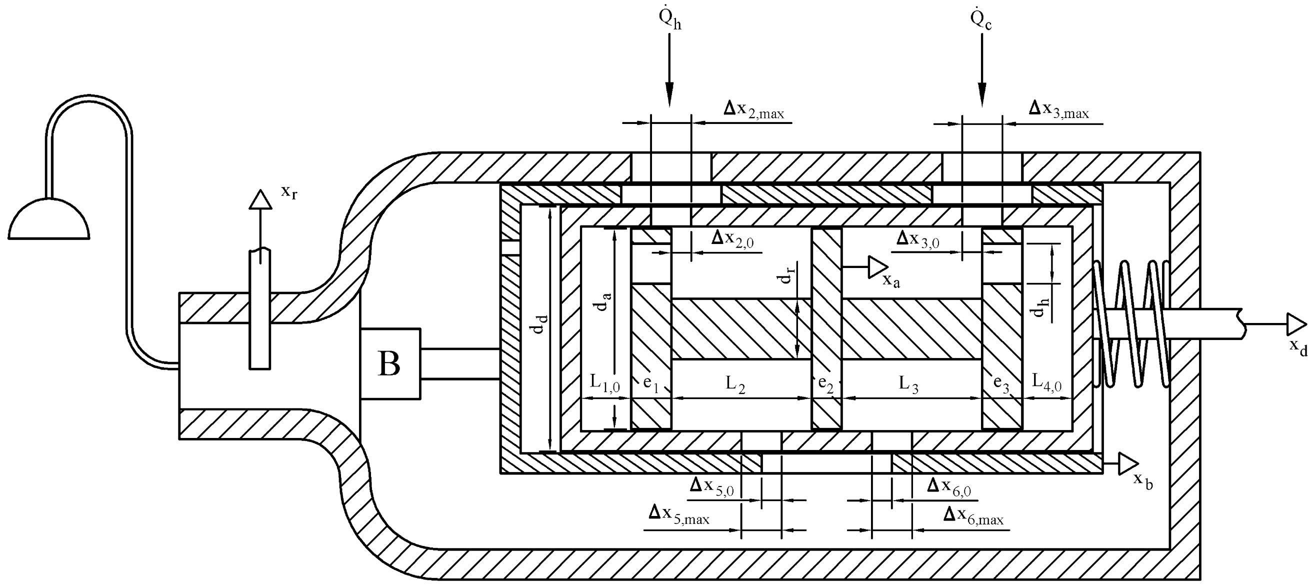

Fig. 2. Schematic representation of the mixer, including its geometrical variables.

desired outlet temperature value of the mixed water, and x_a is the position of the anti-scalding/cold system, as shown in Fig. 2. The sum of Eqs. (7a) and (7b) results in an expression that is null only for an incompressible fluid:

$$-\dot{Q}_{12} + \dot{Q}_{34} = \left[\pi d_a^2/(4\beta)\right]\left[(L_{1,0} + x_a - x_d)(\mathrm{d}P_1/\mathrm{d}t) + (L_{4,0} + x_d + x_a)(\mathrm{d}P_4/\mathrm{d}t)\right]$$

2.2. Pressure drop equations

A fluid flows towards lower pressures, and therefore experiences a pressure drop when flowing in a duct, or from one chamber in the mixer to the next chamber.

Regarding the fluid flow from chamber 1 to 2, and from chamber 3 to 4, assuming that the flow takes place in the laminar regime, which is a reasonable approach given the small velocities involved, the volumetric flow rates $\dot{Q}_{12}$ and $\dot{Q}_{34}$ can be obtained from the Hagen–Poiseille solution as [10]

$$\dot{Q}_{12} = \frac{\pi d_h^4}{128\mu}\frac{(P_1 - P_2)}{e_1} \tag{8a}$$

$$\dot{Q}_{34} = \frac{\pi d_h^4}{128\mu}\frac{(P_3 - P_4)}{e_3} \tag{8b}$$

where d_h is the diameter of the hole and e_1 and e_3 are the thicknesses of the separating discs (the hole depth).

It is assumed that the remaining fluid exchanges occurring among the mixer chambers take place in the turbulent regime and, in this case, the pressure drop depends on the volumetric flow rate, the discharge coefficient C_d and the cross-section area of the passage, A[10].

For the hot water stream, from the inlet to chamber 2, and after from chamber 2 to chamber 5, it can be written that

$$\dot{Q}_h = C_{d,2}A_2\sqrt{\frac{2(P_{h,0} - P_2)}{\rho}} \tag{9a}$$

$$\dot{Q}_{h,out} = C_{d,5}A_5\sqrt{\frac{2(P_2 - P_5)}{\rho}} \tag{9b}$$

For the cold water stream, from the inlet to chamber 3, and after from chamber 3 to chamber 5, the following holds:

$$\dot{Q}_c = C_{d,3}A_3\sqrt{\frac{2(P_{c,0} - P_3)}{\rho}} \tag{10a}$$

$$\dot{Q}_{c,out} = C_{d,6}A_6\sqrt{\frac{2(P_3 - P_5)}{\rho}} \tag{10b}$$

Areas of cross-sections A_2, A_3, A_5 and A_6 can change and they are obtained as

$$A_2 = \Delta x_2 \pi d_a f_1 \tag{11a}$$

$$A_3 = \Delta x_3 \pi d_a f_1 \tag{11b}$$

$$A_5 = \Delta x_5 \pi d_d f_2 \tag{11c}$$

$$A_6 = \Delta x_6 \pi d_d f_2 \tag{11d}$$

where f_1 and f_2 are the fractions of the perimeter corresponding to the ports from one chamber to the next one. For a port of width Δx, which varies with the position of the mobile element under consideration, if all the perimeter of the tubular element of diameter d is used, the area of the cross-section (fluid passage) is $A = \Delta x \pi d$ and it is $f = 1$. If only one-half of the perimeter of the tubular element is used for the fluid passage it is $A = \Delta x \pi d/2$, and then $f = 1/2$.

Distances Δx_2 and Δx_3 change depending on the position of the anti-scalding/cold system and, following Fig. 2, they can be obtained as

$$\Delta x_2 = \begin{cases} 0 & \text{if } \Delta x_{2,0} + x_d - x_a < 0 \\ \Delta x_{2,0} + x_d - x_a & \text{if } 0 \leqslant \Delta x_{2,0} + x_d - x_a \leqslant \Delta x_{2,max} \\ \Delta x_{2,max} & \text{if } \Delta x_{2,0} + x_d - x_a > \Delta x_{2,max} \end{cases} \tag{12a}$$

$$\Delta x_3 = \begin{cases} 0 & \text{if } \Delta x_{3,0} + x_a - x_d < 0 \\ \Delta x_{3,0} + x_a - x_d & \text{if } 0 \leqslant \Delta x_{3,0} + x_a - x_d \leqslant \Delta x_{3,\max} \\ \Delta x_{3,\max} & \text{if } \Delta x_{3,0} + x_a - x_d > \Delta x_{3,\max} \end{cases} \quad (12b)$$

Similarly, distances Δx_5 and Δx_6 change depending on the position of the mixer and, once again from Fig. 2, they can be obtained as

$$\Delta x_5 = \begin{cases} 0 & \text{if } \Delta x_{5,0} + x_d - x_b < 0 \\ \Delta x_{5,0} + x_d - x_b & \text{if } 0 \leqslant \Delta x_{5,0} + x_d - x_b \leqslant \Delta x_{5,\max} \\ \Delta x_{5,\max} & \text{if } \Delta x_{5,0} + x_d - x_b > \Delta x_{5,\max} \end{cases} \quad (13a)$$

$$\Delta x_6 = \begin{cases} 0 & \text{if } \Delta x_{6,0} + x_b - x_d < 0 \\ \Delta x_{6,0} + x_b - x_d & \text{if } 0 \leqslant \Delta x_{6,0} + x_b - x_d \leqslant \Delta x_{6,\max} \\ \Delta x_{6,\max} & \text{if } \Delta x_{6,0} + x_b - x_d > \Delta x_{6,\max} \end{cases} \quad (13b)$$

In the foregoing equations $\Delta x_{2,0}$, $\Delta x_{3,0}$, $\Delta x_{5,0}$ and $\Delta x_{6,0}$ are the reference values for Δx_2, Δx_3, Δx_5 and Δx_6, and $\Delta x_{2,\max}$, $\Delta x_{3,\max}$, $\Delta x_{5,\max}$, and $\Delta x_{6,\max}$ are their maximum respective values, as illustrated in Fig. 2.

At the mixer outlet there exists also a pressure drop, evaluated as

$$\dot{Q}_{out} = C_{d,7} A_7 \sqrt{\frac{2(P_5 - P_6)}{\rho}} \quad (14)$$

where A_7 is the cross-section of the output opening, controlled by the operator, and is expressed as a fraction of the overall available cross-section usually given by a duct of $\frac{1}{2}''$ internal diameter.

According to Merrit [10] and White [11], for an orifice in a plate, for high values of the local Reynolds number, the discharge coefficient C_d is independent of the volumetric flow rate, and it varies between 0.60 and 0.61. The local Reynolds number in a particular passage can be obtained as

$$Re = \rho v \Delta x / \mu \quad (15)$$

where Δx is the smallest dimension of the fluid passage under analysis. The local volumetric flow rate can be obtained as $\dot{Q} = vA = v(f\pi d)\Delta x$, where f is the fraction of the perimeter associated with the passage, and the local Reynolds number can be expressed alternatively as

$$Re = \frac{\rho \dot{Q}}{\mu (f \pi d)} \quad (16)$$

For the usual dimensions in a real mixer, for water, and for $f = 1/2$, it can be obtained that $Re \approx 10^3 \dot{Q}$. For a typical value of $\dot{Q} = 10$ l/min the following holds: $Re \approx 10^4$. Different internal constructions of the valve lead to different resistances to the flow of water, and thus to different overall volumetric flow rates through the mixer. In this work, even if the pathways are not holes in a plate, and the local flow is not turbulent in all the situations, the value $C_d = 0.60$ will be used for all the involved discharge coefficients [10].

Attached to the mixer output other appliances can be installed such as, for example, a shower, which represents an additional pressure drop. This can be expressed as

$$P_6 - P_{atm} = (1/2) K_8 \rho (\dot{Q}_{out}/A_8)^2 \quad (17)$$

where A_8 is the cross-section of the output duct (typically with the internal diameter $\frac{1}{2}''$), and K_8 is the pressure drop coefficient associated with the tube length and possibly other fittings, such as bends and the showerhead.

Atmospheric pressure P_{atm} is taken as $P_{atm} = 0$, and thus all the remaining involved pressures are relative pressures, as usually made when dealing with domestic hydraulic systems.

2.3. Momentum equations

The position of the mixer mobile parts needs to be evaluated at all instants in order to obtain the corresponding volumetric flow rates and the water outlet temperature.

For the anti-scalding/cold system, which moves with some fluid, the momentum equation is

$$(m_a + m_f)\frac{dv_a}{dt} = \frac{\pi(d_a^2 - d_h^2)}{4}(P_1 - P_4) + \frac{\pi d_h^2}{4}(P_2 - P_3) - b v_a \quad (18)$$

where $v_a = dx_a/dt$ and b is the friction coefficient associated with the existing contact between the anti-scalding/cold system and its external cylinder. High values of the friction coefficient b can be used to consider the situation of a thermostatic mixer from which the anti-scalding/cold system is absent (with low mobility relative to the mixer element). In this way, the presented model can be used for both cases of thermostatic mixers with or without the anti-scalding/cold device, as the action of the anti-scalding/cold device can be easily inhibited giving to the friction coefficient b a high value. In this equation, m_a and m_f are the masses of the anti-scalding/cold system and of the involved fluid, respectively, which can be evaluated as

$$m_a = (\rho_a \pi/4)\big[d_a^2(e_1 + e_2 + e_3) + d_r^2(L_2 + L_3) - d_h^2(e_1 + e_3)\big] \quad (19)$$

$$m_f = (\rho_f \pi/4)\big[(d_a^2 - d_r^2)(L_2 + L_3) + d_h^2(e_1 + e_3)\big] \quad (20)$$

Once the mechanical part of the problem has been defined, its thermal part remains to be modeled. As it will be seen, they are strongly linked, and a simultaneous solution is needed to predict the whole behavior of the thermostatic mixer.

2.4. Energy conservation equation

The energy conservation equation, when applied to an open thermodynamic system, with negligible variations in the kinetic and gravitational potential energies gives [9]

$$\frac{dE_{cv}}{dt} = \dot{H} + \sum_{in} \dot{m}h - \sum_{out} \dot{m}h \quad (21)$$

where $\dot{H}$ is the rate of heat received by the mixer from its surroundings, and h is the specific enthalpy. Total energy E is expressed as $E = m(u + 0.5v^2 + gz)$ [9], and in this work only its internal energy term, mu is relevant. In the term dE_{cv}/dt it will be considered that the changes in the internal energy can be taken as similar to the changes in the enthalpy, and this term is taken as $\mathrm{d}E_{cv}/\mathrm{d}t \approx \mathrm{d}(mh)/\mathrm{d}t$. The energy conservation equation to be applied in the present work is

$$\frac{\mathrm{d}}{\mathrm{d}t}(\rho_m V_m h_m + \rho_f V_5 h_f) = \dot{H} + \sum_{in} \dot{m}h - \sum_{out} \dot{m}h \tag{22}$$

where ρ_m is the averaged density of the material of the mixer, and h_m is its corresponding specific enthalpy. As the temperature range is small, the specific enthalpy of the water can be evaluated as $h_f = h_{f,0} + c_f(T - T_0)$, and the specific enthalpy of the material of the mixer is evaluated as $h_m = h_{m,0} + c_m(T - T_0)$, where c_m is its (constant) corresponding averaged specific heat, T_0 is a reference temperature for which $h = h_0$ [9], and c_f is the (constant) specific heat of the water.

Application of the energy conservation equation for the mixer, assuming that volume of chamber 5 is constant and that it is the most relevant volume of the involved chambers, leads to

$$(\rho_m V_m c_m + \rho_f V_5 c_f)\frac{\mathrm{d}T_5}{\mathrm{d}t} = \dot{H} + \sum_{in} \rho_f c_f \dot{Q} T - \sum_{out} \rho_f c_f \dot{Q} T \tag{23}$$

where it is assumed that the temperature of the material of the mixer equals that of the fluid in chamber 5, T_5. This is not strictly the case, but a better modeling needs a very detailed knowledge of the particular inner construction of the mixer. It is also assumed that the small effects due to fluid compressibility are not important in the energy conservation equation.

The heat exchanged between the surroundings and the mixer, $\dot{H}$, can be evaluated as [12]

$$\dot{H} = h_{ext} A_{ext}(T_{amb} - T_5) \tag{24}$$

where h_{ext} is the exterior convection heat transfer coefficient for the mixer, usually subjected to natural convection with still air, A_{ext} is the exterior surface area of the mixer exposed to the environment, and it is assumed that the exterior surface of the mixer is kept at temperature T_5.

Considering that the mixer has two inlet ports and one outlet port, the energy conservation equation gives

$$(\rho_m V_m c_m + \rho_f V_5 c_f)\frac{\mathrm{d}T_5}{\mathrm{d}t} = h_{ext} A_{ext}(T_{amb} - T_5) + \rho_f c_f(\dot{Q}_{h,0} T_h + \dot{Q}_{c,0} T_c - \dot{Q}_{out} T_5) \tag{25}$$

where $\dot{Q}_{h,0}$ and $\dot{Q}_{c,0}$ are the inlet volumetric flow rates of the hot and cold water streams, respectively, both conditioned by the inlet pressures of the hot and cold streams and the internal geometry of the mixer. The relationship to time of the outlet temperature, T_5, will be obtained from this equation.

2.5. Equation for the bulb

Regarding the bulb, the position of its mobile part, x_b, is a function of its temperature T_b, and it can be expressed as

$$x_b = x_{b,0} + \alpha(T_b - T_0) \tag{26}$$

In this equation, T_0 is a reference temperature for which $x_b = x_{b,0}$, and α is the linear expansion coefficient of the bulb. Common values for α are close to 0.25 mm/K. The coefficient α can simply stand for the volumetric expansion coefficient of the liquid or the wax that fills the bulb, or it can be amplified by using a rubber cone in a conical tube [1]. Due to this fact, the assembly of this kind of mixer is carried out in a temperature controlled environment, at temperature T_0.

The bulb temperature is related to the outlet (mixture) fluid temperature, and the energy balance for the bulb gives [12]

$$m_b c_b \frac{\mathrm{d}T_b}{\mathrm{d}t} = h_b A_b(T_5 - T_b) \tag{27}$$

where h_b is the convection heat transfer coefficient between the fluid and the bulb, and A_b is the area of its wetted surface. The convection heat transfer coefficient h_b can be obtained by using some available correlations for heat transfer from or to cylinders in cross flow, through an expression of the type [12]

$$h_b = C\,Re^m\,Pr^{1/3}\,k_f/d_b \tag{28}$$

In the foregoing equations, Pr is the Prandtl number, Re is the Reynolds number based on the bulb diameter, k_f is the fluid thermal conductivity, and C and m are parameters dependent on the Reynolds number [12]. The value obtained for the convection heat transfer coefficient will be strongly dependent on the particular mixer geometry and on the flow around the bulb. An expression for its evaluation, for water at room temperature, can be

$$h_b = 1.87 \times 10^5\,\dot{Q}_{out}^{0.54}\ \mathrm{W/(m^2\,K)} \tag{29}$$

2.6. Equation for the temperature control lever

The user sets the mixer on the desired temperature by moving the temperature control lever, thus setting the position x_d. The desired temperature and the lever position are linearly related through the expression

$$x_d = x_{d,0} + \gamma(T_d - T_0) \tag{30}$$

Usually, this lever consists of a screw with a given fixed pitch. If, for example, the screw has a 5 mm pitch, and it is to obtain the temperature range 20–60 °C in one turn, then $\gamma = 5/40$ mm/K.

3. Steady-state solution

When the mixer operates in steady-sate conditions, with constant inlet pressures and temperatures, and when there are no changes in the flow and temperature control levers, the foregoing model can be simplified and its steady-state solution can be obtained.

The energy conservation equation, Eq. (25), can be written as

$$T_{5,ss} = \frac{\dot{Q}_{h,0} T_h + \dot{Q}_{c,0} T_c + [h_{ext} A_{ext}/(\rho_f c_f)] T_{amb}}{\dot{Q}_{h,0} + \dot{Q}_{c,0} + [h_{ext} A_{ext}/(\rho_f c_f)]} \tag{31}$$

given that, under these conditions, $\dot{Q}_{h,out} = \dot{Q}_{h,0}$, $\dot{Q}_{c,out} = \dot{Q}_{c,0}$ and $\dot{Q}_{out} = \dot{Q}_{h,0} + \dot{Q}_{c,0}$. It is to be noted that, in

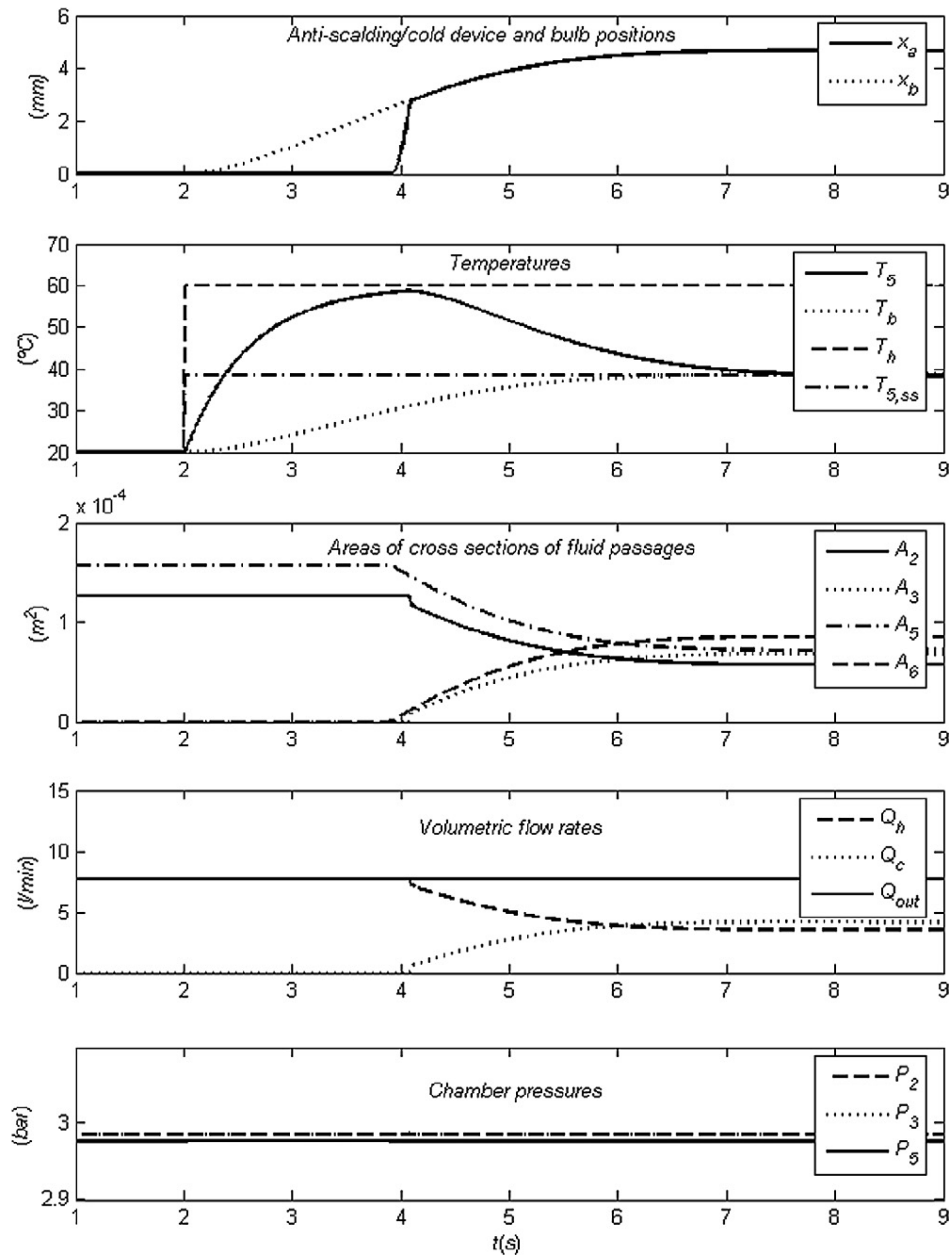

Fig. 3. Response of the mixer when an increasing step in the hot temperature is applied, from 20 to 60 °C. [$T_d = 38$ °C, $T_c = 20$ °C, $x_r = 50\%$, $P_h = 3$ bar, $P_c = 3$ bar, $T_{5ss} = 38.6$ °C (steady state).]

steady-state conditions, the anti-scalding/cold device reaches an equilibrium position, thus given that $P_2 = P_3$, and Eqs. (9b) and (10b) give $\dot{Q}_{h,out} = C_{d,5}A_5\sqrt{2\Delta P/\rho_f}$ and $\dot{Q}_{c,out} = C_{d,6}A_6\sqrt{2\Delta P/\rho_f}$, with $\Delta P = (P_2 - P_5) = (P_3 - P_5)$. If all the discharge coefficients, C_d, are made equal, the energy conservation equation gives

$$T_{5,ss} = \frac{A_5T_h + A_6T_c + \left[\frac{h_{ext}A_{ext}}{\rho_f c_f}\frac{1}{C_d}\sqrt{\frac{\rho_f}{2\Delta P}}\right]T_{amb}}{A_5 + A_6 + \left[\frac{h_{ext}A_{ext}}{\rho_f c_f}\frac{1}{C_d}\sqrt{\frac{\rho_f}{2\Delta P}}\right]} \quad (32)$$

In the limiting situation of $h_{ext}A_{ext} \to 0$ (when the mixer is not exchanging heat with the surroundings), the output temperature is the weight averaged temperature of the inlet hot and cold temperatures, and the weighting factors are the areas of the passages of the hot and cold streams. For a theoretical mixer with $h_{ext}A_{ext} \to +\infty$, the output temperature will tend to the ambient temperature, as the heat exchange with the environment is the dominant mechanism in this case.

Eq. (32) can be worked out, with areas A_5 and A_6 given by Eqs. (11c) and (11d), and $\Delta x_5 = \Delta x_{5,0} + x_d - x_b$ and $\Delta x_6 = \Delta x_{6,0} + x_b - x_d$ as given by Eqs. (13a) and (13b), respectively, to give

$$\begin{aligned} T_{5,ss} = T_d + \big[&(T_c - T_d) + (R_1 - R_2)(T_0 - T_d) \\ &+ R_3(T_h - T_c) + R_4(T_{amb} - T_d)\big] \\ &/(1 + R_1 + R_4) \end{aligned} \quad (33)$$

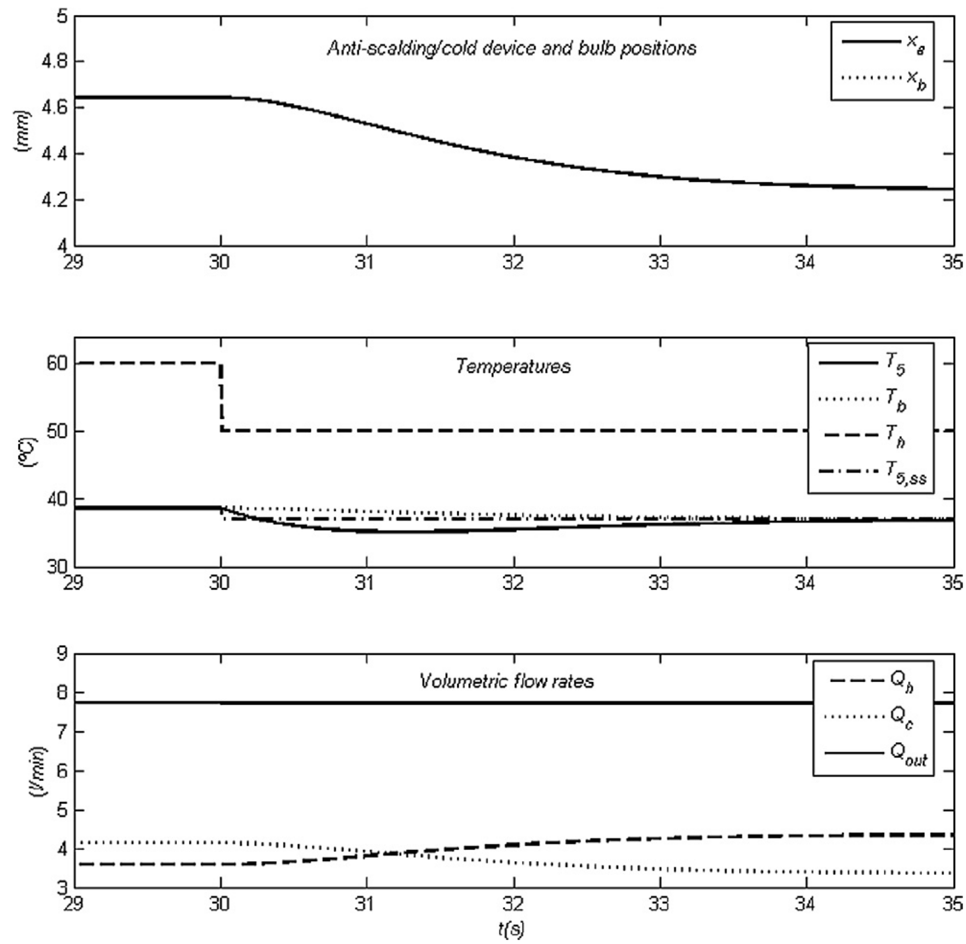

Fig. 4. Response of the mixer when a decreasing step in the hot temperature is applied, from 60 to 50 °C. ($T_d = 38$ °C, $T_c = 20$ °C, $x_r = 50\%$, $P_{h,0} = 3$ bar, $P_{c,0} = 3$ bar.)

where the dimensionless parameters R_1, R_2, R_3 and R_4 are defined as

$$R_1 \equiv \frac{\alpha(T_h - T_c)}{\Delta x_{5,0} + \Delta x_{6,0}} \tag{34}$$

$$R_2 \equiv \frac{\gamma(T_h - T_c)}{\Delta x_{5,0} + \Delta x_{6,0}} \tag{35}$$

$$R_3 \equiv \frac{\Delta x_{5,0} + x_{d,0} - x_{b,0}}{\Delta x_{5,0} + \Delta x_{6,0}} \tag{36}$$

$$R_4 \equiv \frac{h_{ext} A_{ext}}{\rho_f c_f} \frac{1}{C_d} \sqrt{\frac{\rho_f}{2\Delta P}} \frac{1}{\pi d_d f_2} \frac{1}{\Delta x_{5,0} + \Delta x_{6,0}} \tag{37}$$

For the thermostatic mixer that is not exchanging heat with the ambient, that is, with $h_{ext} A_{ext} \to 0$, Eq. (33) applies with $R_4 = 0$.

The main point of Eq. (33) is that, in steady-state conditions, the outlet mixture temperature T_5 is different from the desired temperature T_d, and the term in the fraction of the right-hand side of Eq. (33) can be seen as the deviation from the desired temperature value. Only for some combinations of the involved parameters (typically non-realistic situations) will be $T_5 = T_d$. If this is true for steady-state conditions, it will be true also for transient conditions and, in general, the output mixture temperature will be different from the desired temperature set by the operator, even if this difference is small. In this work, some studies will be made in order to assess how the different parameters affect the mixture temperature deviation from its desired value, both under steady-state conditions and under transient conditions.

4. Numerical modeling

Regarding the numerical model, a Simulink block diagram integrating the Modelica model of the thermostatic mixer was developed to simulate the thermostatic mixer with different inputs. In order to reduce the system stiffness, introduced with possible fast changes in the input test signals, all the inputs are introduced with a smooth continuous trajectory modeled by a third order polynomial. The hot water pressure, $P_{h,0}$, for example, is generated with $P_{h,0} = a \cdot t^3 + b \cdot t^2 + c \cdot t + d$. The parameters a, b, c and d depend on the start and end points and the initial and final values of the derivative of the variable being controlled, as well as the total time to complete the trajectory. The trajectory is expressed as a function of time (t) in seconds. This third order polynomial generates gentle, smooth curves that reduce the overall model stiffness. The numerical simulations use the variable step “ode23tb” Matlab solver that is suitable to the numerical integration of stiff systems [13]. This

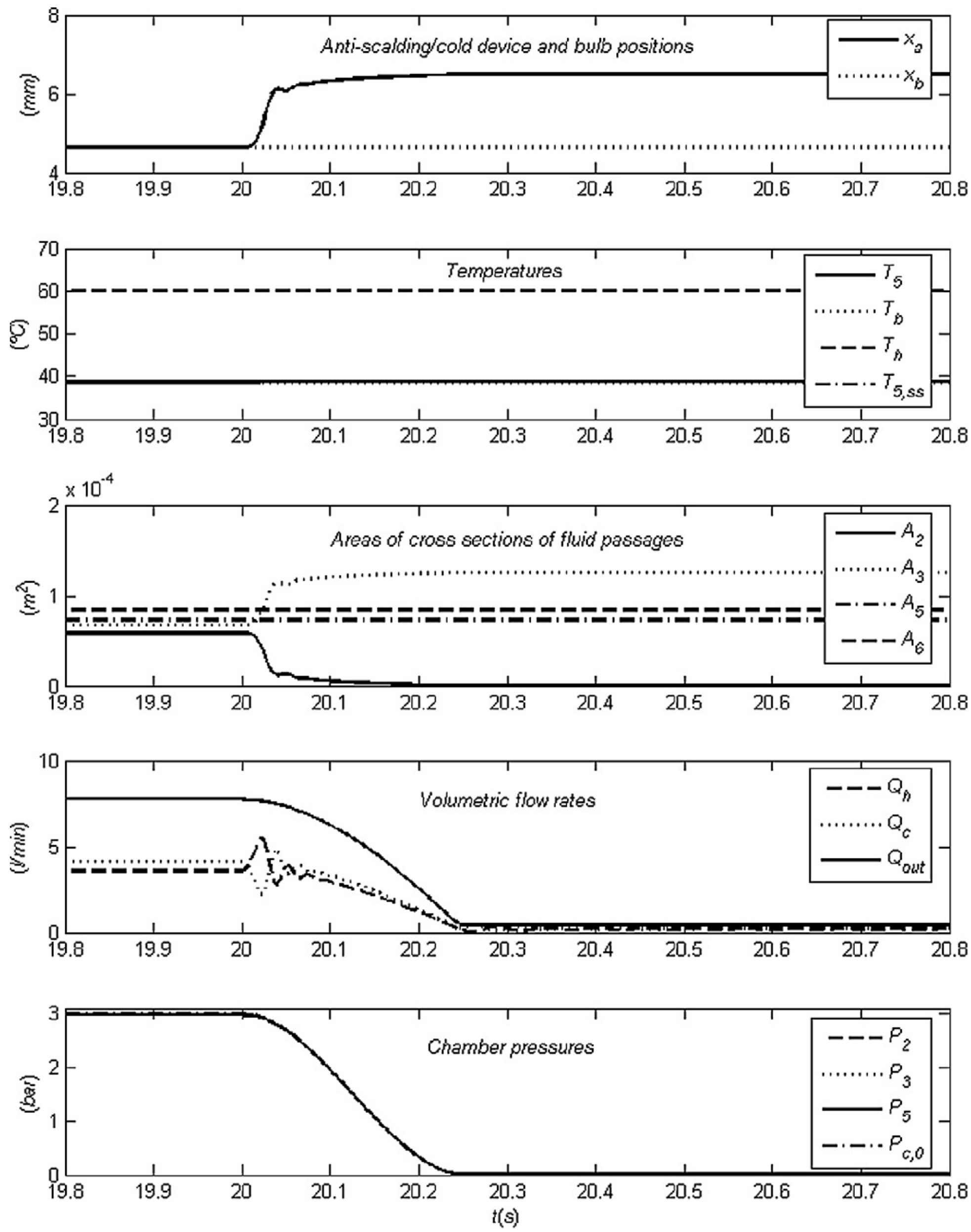

Fig. 5. Response of the valve when a sudden decrease on the inlet pressure of the cold stream is applied, from 3 to 0 Pa (relative). ($T_d = 38\,°C$, $T_c = 20\,°C$, $x_r = 50\%$, $P_{h,0} = 3$ bar.)

solver is an implementation of TR-BDF2, an implicit Runge–Kutta formula with a first stage that is a trapezoidal rule step and a second stage that is a backward differentiation formula of order two.

5. Illustrative results

Some results are presented showing the capabilities of both the physical model and of the numerical model.

The parameters considered are set out in Table 1; some of them can assume different values only if locally and explicitly specified in the text.

Fig. 3 presents the valve response to changes in the hot water inlet temperature, whose value suddenly changes from 20 to 60 °C. This sudden change on the inlet temperature of the hot water stream induces changes in the valve that need nearly 5 s to occur. It is observed that the outlet temperature T_5 increases with time during nearly 2 s, reaching then the maximum value of the inlet hot water temperature, 60 °C, hence creating a risk of scalding. After that, the output water temperature decreases during nearly 3 s, and reaches the expected steady-state value of 38.6 °C, which differs only by 0.6 °C from the desired temperature value, $T_d = 38$ °C. Potential risk for scalding exists for nearly 2 s, the period during which the outlet temperature exceeds 50 °C. In regards to the volumetric flow rate, it is observed that its initial value is given mainly by the hot water stream (when the hot water stream is at the temperature of 20 °C), and that the cold inlet water begins contributing to the

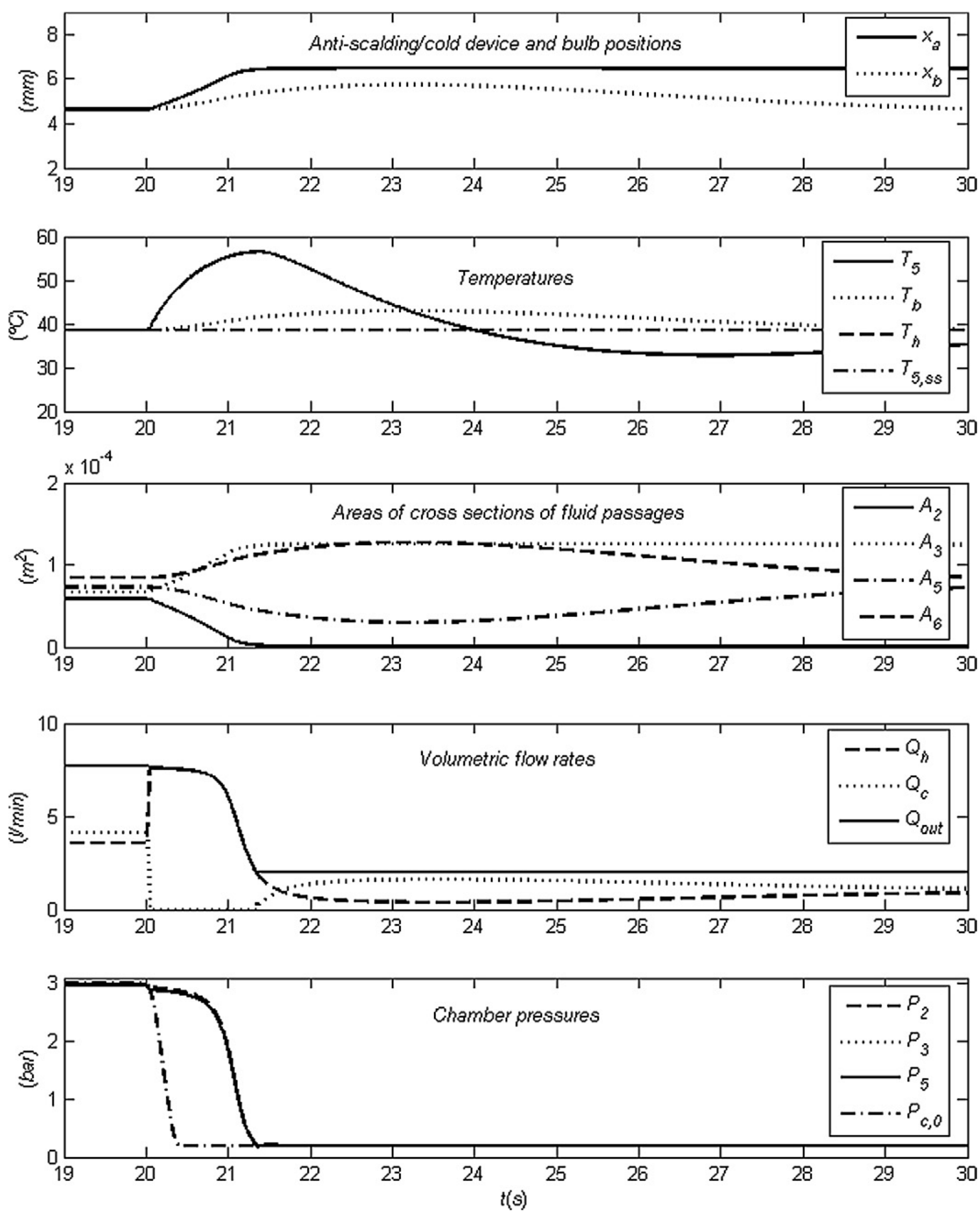

Fig. 6. Response of the valve to a sudden decrease on the inlet pressure of the cold stream, from 3 to 0 bar, and a high value of the friction coefficient, $b = 1000$ N s/m ($T_d = 38\,^\circ$C, $T_c = 20\,^\circ$C, $x_r = 50\%$, $P_{c,0} = 3$ bar.)

overall water flow rate 2 s after the sudden change in the hot water inlet temperature. It is observed that, for the imposed internal geometry of the valve, with $x_r = 0.5$, the inlet pressure of 3 bar of both the hot and cold streams leads to an overall flow rate of nearly 7.5 l/min. When the steady-state conditions are reached, the cold water inlet has a slightly higher contribution than the hot water inlet to the overall volumetric flow rate. There are no noticeable changes on the pressure of the involved streams, including the output pressure, due to this sudden change on the inlet hot water temperature.

Fig. 4 plots the response of the valve to a sudden decrease of the hot water inlet temperature, from 60 to 50 °C. Once again it is seen that this modification induces changes that need nearly 5 s to take place. Output temperature decreases below the expected steady-state temperature, during nearly 4 s, after which the steady state conditions are reached. It is to be noted that the steady state temperature value changes with the temperatures of the inlet streams, as given by Eq. (33). It is also observed that, initially, the contribution of the hot water volumetric flow rate is higher than the cold water flow rate, and that the inverse situation corresponds to the new equilibrium operating conditions of the valve. Pressures in the chambers of the valve keep the values of $P_2 = P_3 = 2.986$ bar, and $P_5 = 2.976$ bar, which are not affected by a sudden temperature change induced on the hot water inlet.

Fig. 5 shows the valve response to a sudden decrease on the cold water inlet pressure from 3 to 0 bar. This variation induces a fast change on the anti-scalding/cold device position, and the

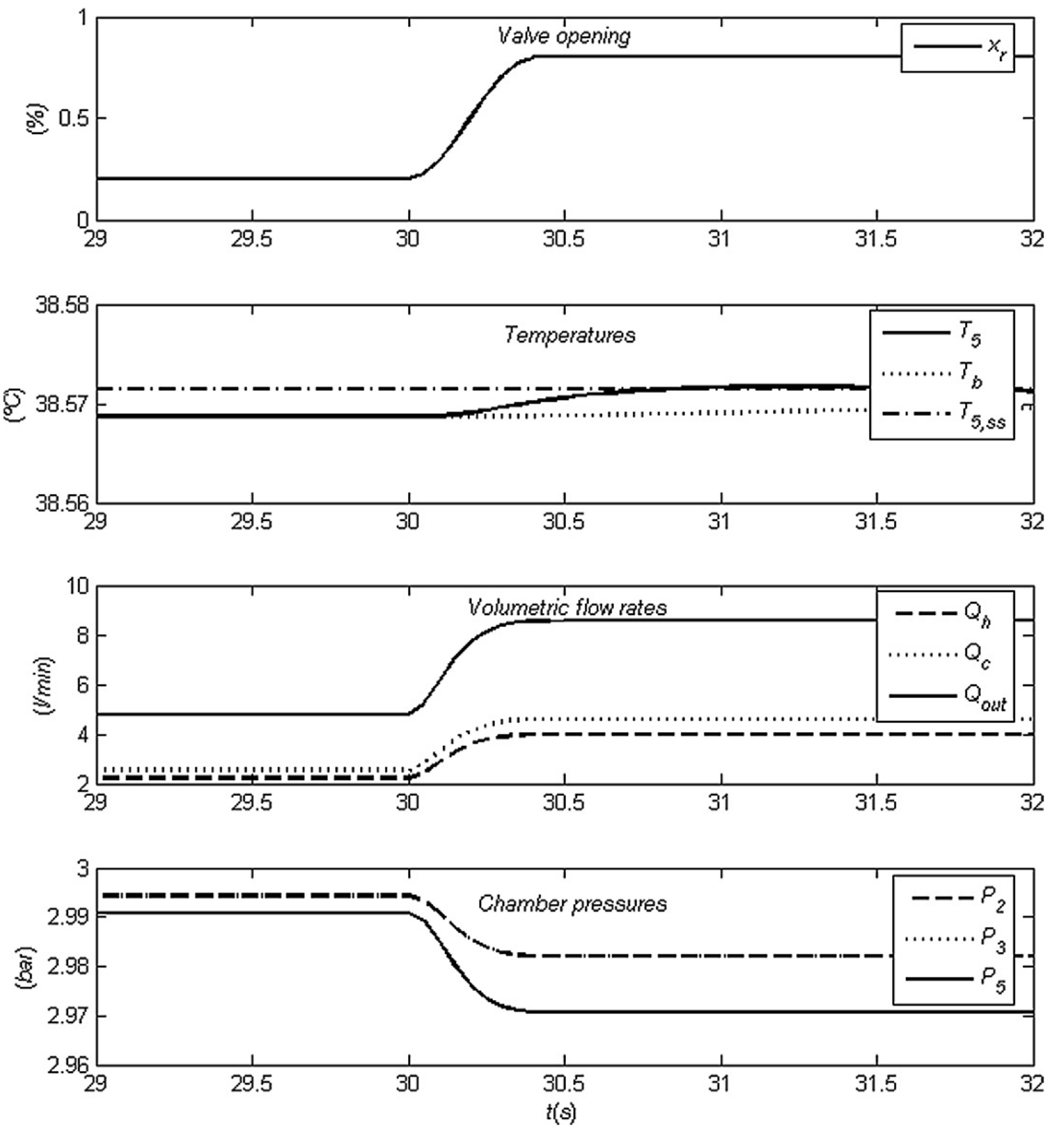

Fig. 7. Response of the mixer to a change on the opening, x_r. ($T_h = 60\,^\circ\mathrm{C}$, $T_c = 20\,^\circ\mathrm{C}$, $P_{h,0} = 3$ bar, $P_{c,0} = 3$ bar.)

output volumetric flow rate drops to a value close to zero in the very short time period of 0.25 s. The output temperature remains unchanged; as the valve loses heat to the environment, there is a small hot water volumetric flow rate entering the valve, releasing heat, and therefore keeps the valve at the expected steady-state temperature. Regarding the hot and cold volumetric flow rate contributions, Fig. 5 allows the observation of a behavior similar to a damped oscillation, during a short time period, with the hot and cold water streams alternating in importance to the overall volumetric flow rate. A relevant conclusion taken from Fig. 5 is that the anti-scalding/cold device is working very well considering its main function, which is to reduce the valve response time when sudden decreases on the inlet pressures of one stream occur.

Fig. 6 presents a similar situation to Fig. 5 but using a high value for the friction coefficient, which is equivalent to have the anti-scalding/cold system acting very slowly. In this case, the mixer response time is delayed, and relevant changes occur on the outlet temperature. When the inlet cold stream pressure decreases, the hot stream is dominant and the outlet temperature reaches a value close to the inlet hot water stream and creates a risk of scalding. Nearly 5 s after the pressure decrease, the outlet temperature decreases during nearly 5.5 s, and reaches a minimum some degrees below the desired temperature, and again starts rising to the expected steady-state value. When the inlet pressure of the cold stream decreases, a noticeable reduction on the outlet volumetric flow rate from nearly 8 l/min to nearly 2 l/min is also observed. It is thus concluded that the anti-scalding/cold system needs to have a low friction (a nearly free possibility of displacement) in order to give a short response time to the mixer when the inlet pressure of the hot or cold streams suddenly drops.

Fig. 7 shows the valve response to a variation on the valve opening from $x_r = 20\%$ to $x_r = 80\%$. The major changes are observed on the output volumetric flow rate, which changes from nearly 4.5 l/min to nearly 8.5 l/min, as an overall contribution of both the hot and cold inlet water streams. The increase on the valve opening results in higher volumetric flow rates, be-

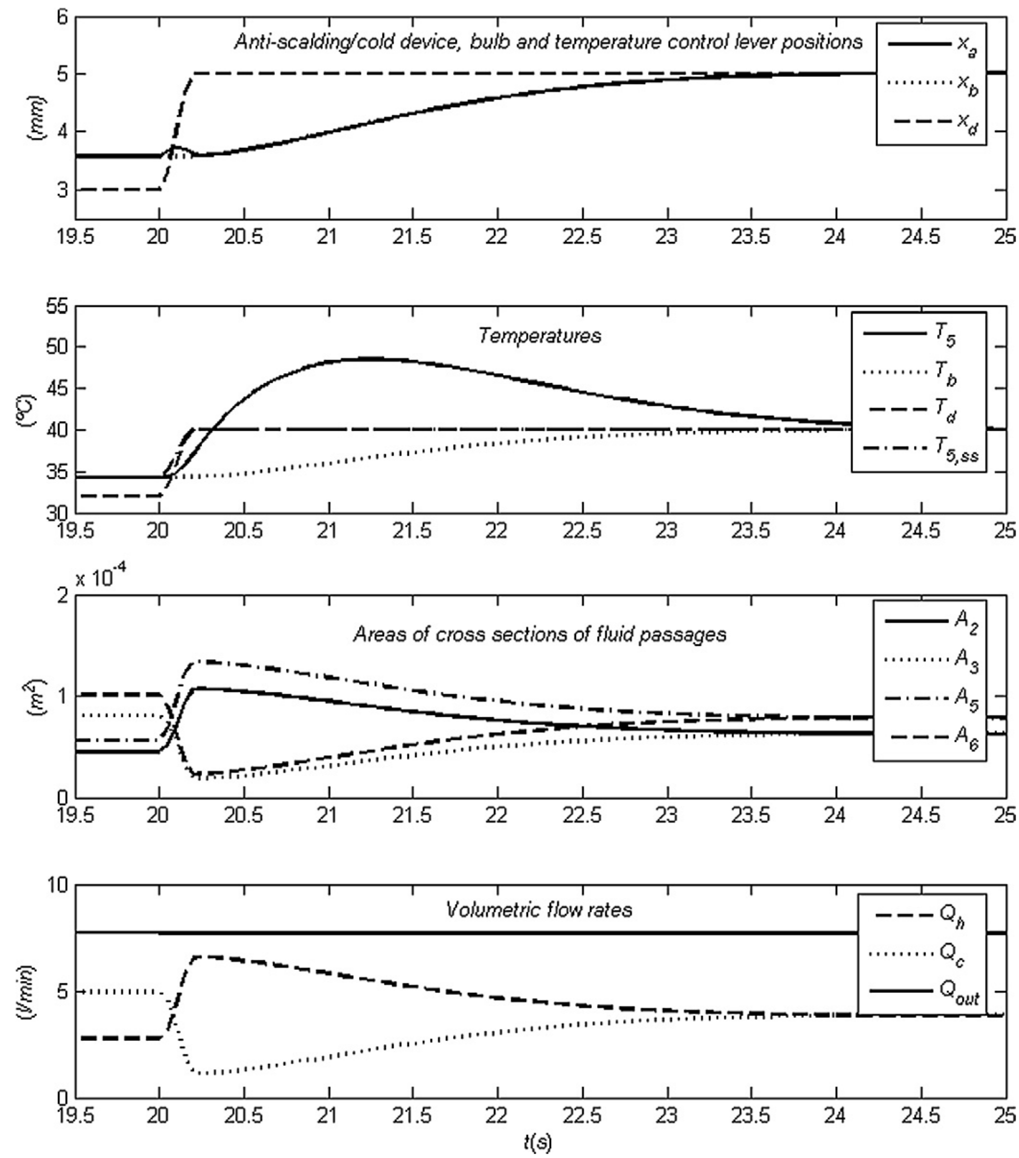

Fig. 8. Response of the mixer to a change on the desired temperature values from 32 to 40 °C. ($T_h = 60$ °C, $T_c = 20$ °C, $x_r = 50\%$, $P_{h,0} = 3$ bar, $P_{c,0} = 3$ bar.)

ing the overall pressure drop kept at its reference value of 3 bar. Regarding the temperature, no noticeable changes are observed after the imposed change on the valve opening. The positions of the anti-scalding/cold system and of the bulb remain essentially constant ($x_a = 4.64$ mm, $x_b = 4.64$ mm).

Fig. 8 shows the valve response to a change on the desired temperature value from 32 to 40 °C. The effects of this change are observed during nearly 5 s. The output temperature considerably increases to nearly 48 °C in 1.5 s and then slowly decreases to the new expected steady state value in nearly 3.5 s. The volumetric flow rate contribution of the cold water stream is higher before the temperature change, and on the steady-state conditions the contributions of the hot and cold streams are nearly the same. The observed changes on the contributions of the inlet water streams to the overall volumetric flow rate occur without any noticeable change on the outlet volumetric flow rate, which is maintained at nearly 8 l/min. No noticeable changes on the pressures are observed, and are kept about $P_2 = P_3 = 2.986$ bar and $P_5 = 2.976$ bar.

Fig. 9 shows how the characteristics of the bulb affect the valve response time when subjected to a sudden increase in the hot water temperature inlet from 20 to 60 °C. From Eq. (27), the bulb response time is given by $m_b c_b/(h_b A_b)$, which is thus proportional to the dimensional factor $m_b c_b/A_b$. Fig. 9 presents the valve response for different values of this dimensional parameter. Increasing values of this parameter lead to longer valve response time, and to an increase of potential risks of scalding/cold for the user. A compromise exists: larger bulbs, with higher values of the dimensional factor $m_b c_b/(h_b A_b)$, will provide greater displacements under temperature influence, and are thus better for temperature control. However, due to their higher masses, they have higher response times, and they will give a poor performance to the mixer when operating under unsteady conditions.

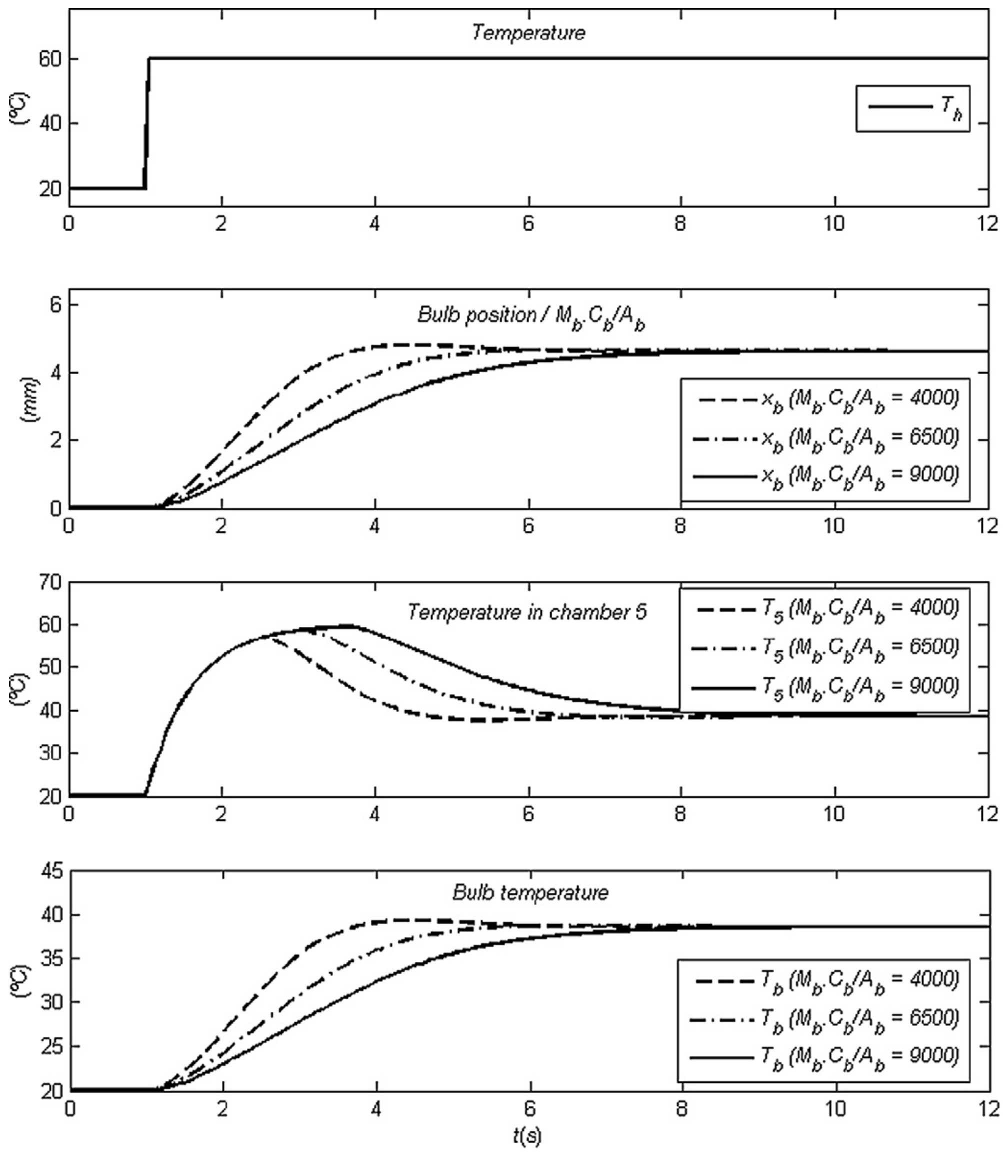

Fig. 9. Influence of the thermal properties of the bulb, through the dimensional factor $m_b c_b/A_b$, when a change of the inlet hot stream exists, from 20 to 60 °C. ($T_d = 38$ °C, $T_c = 20$ °C, $x_r = 50\%$, $P_{h,0} = 3$ bar, $P_{c,0} = 3$ bar.)

Table 1
Numerical values used for governing parameters

$A_b = 17 \times 10^{-4}$ m^2	$A_{ext} = 345 \times 10^{-4}$ m^2	$b = 0.1$ N s/m
$C_b = 600$ J/(kg K)	$C_{p,f} = 4190$ J/(kg K)	$C_{p,steel} = 450$ J/(kg K)
$d_a = 0.02$ m	$d_h = 0.005$ m	$d_r = 0.005$ m
$d_t = 0.00635$ m	$D_d = 0.025$ m	$e_1 = 0.004$ m
$e_2 = 0.003$ m	$e_3 = 0.004$ m	$h_{ext} = 5$ W/(m^2 K)
$L_{10} = 0.005$ m	$L_2 = 0.014$ m	$L_3 = 0.014$ m
$L_{40} = 0.005$ m	$m_b = 0.0185$ kg	$T_{amb} = 20$ °C
$T_0 = 20$ °C	$V_5 = 60 \times 10^{-6}$ m^3	$V_{steel} = 20 \times 10^{-6}$ m^3
$x_{b,0} = 0$ m	$x_{d,0} = -0.002$ m	$\alpha = 0.25 \times 10^{-3}$ m/K
$\beta_e = 2.4 \times 10^9$ Pa	$\gamma = 2.5 \times 10^{-4}$ m/K	$\Delta_{x2,0} = 0.002$ m
$\Delta_{x2,max} = 0.004$ m	$\Delta_{x3,0} = 0.002$ m	$\Delta_{x3,max} = 0.004$ m
$\Delta_{x5,0} = 0.002$ m	$\Delta_{x5,max} = 0.004$ m	$\Delta_{x6,0} = 0.002$ m
$\Delta_{x6,max} = 0.004$ m	$\mu = 6.82 \times 10^{-4}$ kg/(m s)	$\rho = 1000$ kg/m^3
$\rho_{steel} = 7800$ kg/m^3		

Fig. 10 describes the influence of the bulb's linear expansion parameter α on the valve response when a change on the inlet hot water temperature exists, from 20 to 60 °C. This parameter has no influence on the valve response time, but rather on the position reached by the mobile element linked to the bulb. It can be seen that small values of this parameter lead to too small displacements of the valve temperature regulator, which can never reach the required position corresponding to the desired outlet temperature. This is information of great value at the design stage, as no prototypes need to be built with such a bulb, as it never meets the desired performance. A bulb with a small value for α can have a good performance on a mixing valve only if the opening of the mixer, $\Delta x_{5,max} + \Delta x_{6,max}$, is set in accordance with such a value of α. However, small values of α can lead to operation problems of the mixer due to solid particles contained in water, therefore requiring the use of filters, which on the other hand introduce greater pressure drops and maintenance needs, and allows only lower volumetric flow rates.

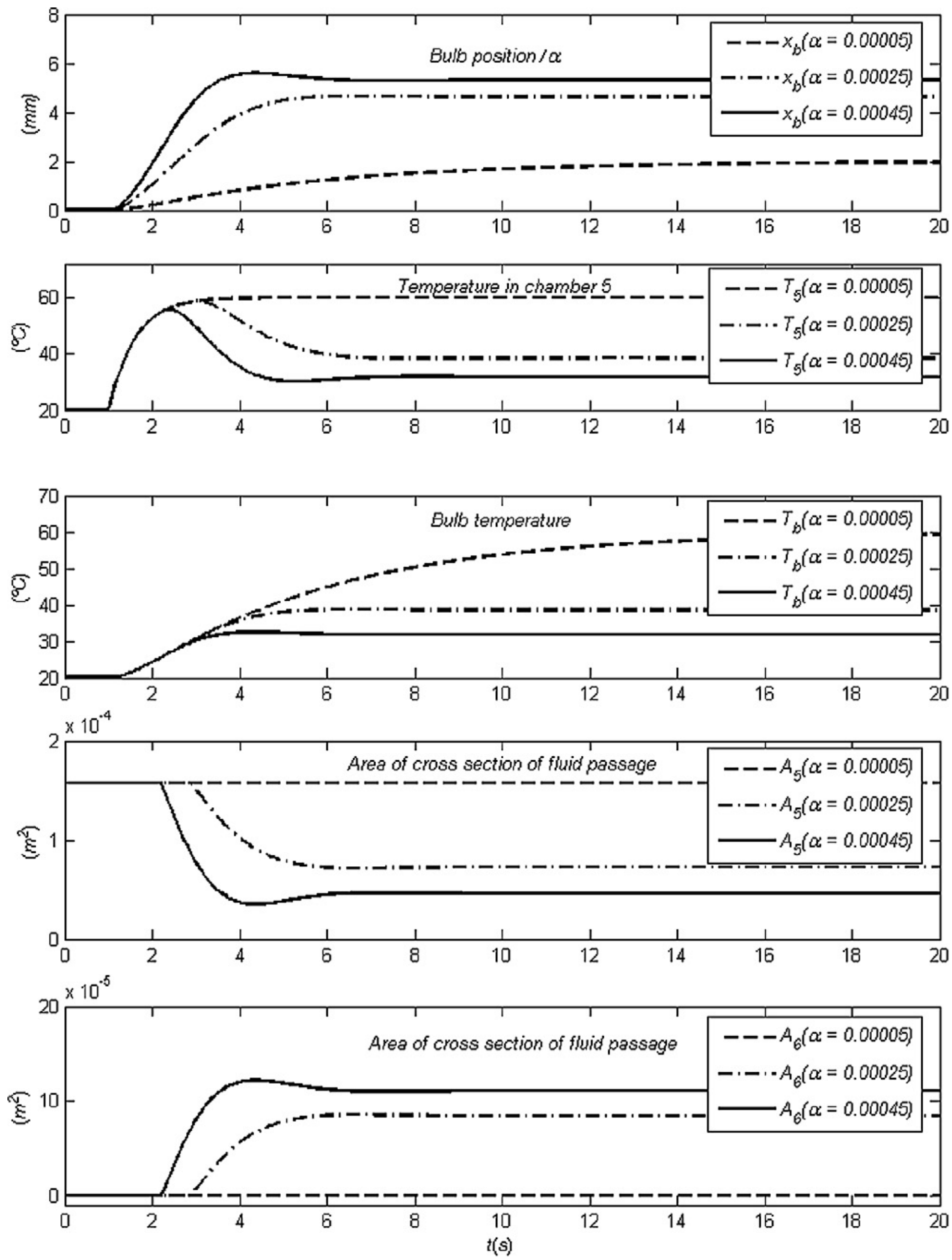

Fig. 10. Behavior of the mixer for different values of parameter α, when a change on the inlet hot water temperature exists, from 20 to 60 °C. ($T_d = 38\,^\circ$C, $T_c = 20\,^\circ$C, $x_r = 50\%$, $P_{h,0} = 3$ bar, $P_{c,0} = 3$ bar.)

6. Conclusions

The modeling of thermal equipment and systems has been shown to be effective. It describes the main parameters and variables, characterizing their individual behavior and relationships to predict the equipment performance under real operating conditions. In the present case, modeling allows a complete understanding about thermostatic mixers, their components, the individual behavior of each component alone, and the behavior of the assembled set. Simulation predicts the behavior of the mixer, when operating both in transient or steady-state conditions and analysis of results gives valuable information to be considered at the development and design phases of this kind of device.

Especially important is the dynamic response of the mixer when subjected to sudden changes on the input streams conditions or/and on the control levers, which enables assessment of the conditions under which scalding or cold shock exist.

A steady-state solution can be analytically obtained, leading to important conclusions, both regarding the output mixture temperature and the main operating characteristics of the mixer. The analytical steady state solution is also important to assess the results of the complete model when simulating the steady-state operation.

It was observed that the anti-scalding/cold system gives the mixer a very short response time when inlet pressures of the hot or cold streams decrease considerably and suddenly. The anti-scaling/cold system response time is considerably shorter than the bulb response time, provided that system can move freely. If friction is considerable, as typically found in a mixer without the anti-scalding/cold device, its response is delayed and is not adequate. For a mixer without the anti-scalding/cold system some potential for scalding/cold exists, what can be crucial when the thermostatic mixer is to be used by children, old people, handicapped people, or others with reduced reaction capacity.

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